

Higgs Field Bubble Theory- A Weak-Field Formulation of Gravity and Light Propagation

Manuscript Version – July 2026

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Abstract

This paper develops the weak-field gravitational sector of the Higgs Field Bubble Theory (HFBT). In this framework, the Higgs field is treated as a continuous physical medium, and matter is interpreted as localised regions of reduced Higgs-field density (depletion bubbles). Gravity arises from spatial gradients in this depletion field rather than from a fundamental force or the curvature of spacetime.

Starting from an energy minimisation principle, a scalar depletion field is introduced and shown to satisfy a Poisson-type field equation. By identifying the gravitational potential with the depletion field, the Newtonian inverse-square law is recovered exactly in the weak-field limit.

Light propagation is described as wave motion through a Higgs medium whose local density determines the local propagation speed. This naturally leads to an effective refractive index that reproduces the standard weak-field expressions for gravitational lensing, Snell-type refraction, and the Shapiro time delay.

The formulation presented here is explicitly restricted to weak, static gravitational fields. Strong-field phenomena, nonlinear Higgs dynamics, black-hole physics, and quantum effects lie outside the scope of the present work and will be addressed separately. The objective of this paper is to establish a mathematically consistent classical foundation for the development of the broader HFBT framework.

1. Introduction

Modern gravitational theory is remarkably successful. Newtonian gravity accurately describes most weak-field phenomena, while General Relativity extends this description to relativistic systems by accounting for the curvature of spacetime. Both theories have received extensive experimental confirmation.

Despite this success, neither theory identifies a physical medium responsible for gravitational interactions. General Relativity describes gravity geometrically through the spacetime metric, while Newtonian gravity treats it as an attractive force between masses. Neither formulation provides a microscopic physical mechanism from which gravitational behaviour emerges.

The Higgs Field Bubble Theory (HFBT) explores an alternative interpretation. It proposes that the Higgs field constitutes a continuous physical medium permeating all space and that matter corresponds to localized regions in which the Higgs field is partially depleted. Within this framework, gravity is interpreted as the natural response of the surrounding Higgs medium to spatial variations in field density.

The purpose of the present paper is not to replace General Relativity. Instead, it seeks to demonstrate that the observed weak-field behaviour of gravity can be recovered from a physically motivated field model based upon Higgs-field depletion.

Specifically, this paper develops four principal results:

1. A scalar Higgs depletion field is introduced to describe localised reductions in Higgs-field density.
2. A governing field equation is derived from an energy minimisation principle.
3. The Newtonian gravitational potential and inverse-square law are recovered exactly in the weak-field limit.
4. Light propagation through the resulting Higgs density gradient is shown to reproduce the standard weak-field results for gravitational lensing and gravitational time delay.

Throughout this work the analysis is restricted to weak, static gravitational fields where linear approximations remain valid. The formulation does not attempt to describe strong gravitational fields, relativistic collapse, quantum behaviour, or cosmological evolution. Those topics require nonlinear extensions of the present theory and will be developed separately.

Within the broader HFBT programme currently under development, gravity represents one sector of a larger field theory. The present work concerns only the Higgs density field responsible for mass and gravitation. Complementary developments suggest that charge, magnetism, spin, and quantum phenomena arise from an independent Higgs phase field coupled to the density field at particle boundaries. Those developments lie beyond the scope of this paper but provide the theory's long-term direction.

2. Higgs Field Depletion Model

2.1 The Higgs Field as a Continuous Medium

The Higgs Field Bubble Theory begins from the assumption that the Higgs field is a continuous physical medium permeating all of space. In its undisturbed state, the field possesses a uniform equilibrium density

$$\rho_{H0}.$$

Matter is interpreted as localized regions in which this equilibrium density is reduced. Rather than treating elementary particles as point-like objects, HFBT models them as stable depletion structures embedded within the Higgs medium.

The degree of local depletion is described by the dimensionless field variable

$$\nu(\mathbf{x}) = \frac{\rho_{H0} - \rho_H(\mathbf{x})}{\rho_{H0}},$$

where

- $\rho_H(\mathbf{x})$ is the local Higgs-field density,
- ρ_{H0} is the undisturbed Higgs-field density.

The depletion field therefore satisfies

$$0 \leq \nu \leq 1.$$

with

$$\nu = 0$$

representing undisturbed Higgs field,

and

$$\nu > 0$$

representing regions of Higgs-field depletion associated with matter.

In the weak-field regime considered here,

$$\nu \ll 1,$$

allowing linear approximations throughout the analysis.

2.2 Physical Interpretation

The quantity $v(\mathbf{x})$ is the fundamental gravitational variable of the theory.

Unlike the Newtonian potential, which is introduced as a mathematical potential function, the depletion field represents a physical property of the underlying medium: the fractional reduction in Higgs-field density.

Spatial variations in this depletion field produce gradients within the Higgs medium. It is these gradients that generate the observed gravitational behaviour.

Within HFBT, gravity is therefore interpreted as an emergent phenomenon arising from the tendency of the Higgs field to restore its equilibrium density.

This interpretation provides a direct physical mechanism while remaining mathematically compatible with the observed weak-field behaviour of gravity.

3. Energy Principle

3.1 Motivation

Most classical field theories are obtained by minimising an appropriate energy functional.

Examples include

- electrostatics,
- elasticity,
- diffusion,
- and General Relativity through the Einstein–Hilbert action.

Following the same philosophy, HFBT assumes that the Higgs field adopts the configuration of minimum total energy consistent with the surrounding matter distribution.

The gravitational field equation therefore emerges as the Euler–Lagrange equation associated with an energy functional rather than being introduced as an independent postulate.

3.2 Energy Functional

The simplest rotationally invariant energy functional consistent with the weak-field assumptions is

$$E[v] = \int \left[\frac{A}{2} |\nabla v|^2 - B \mu(\mathbf{x}) v \right] d^3x,$$

where

- A is the Higgs-field stiffness constant,
- B describes the coupling between matter and the Higgs field,
- $\mu(\mathbf{x})$ is the ordinary matter density,
- $v(\mathbf{x})$ is the Higgs depletion field.

The first term represents the elastic energy stored in spatial variations of the Higgs field.

The second term represents the interaction between matter and the surrounding Higgs medium. Regions containing matter lower the local field density and therefore act as sources of depletion.

3.3 Dimensional Consistency

The two terms of the energy functional possess identical physical dimensions.

Since

$$[v] = 1,$$

The gradient has units

$$[\nabla v] = \text{m}^{-1},$$

requiring

$$[A] = \text{J m}^{-1}$$

so that

$$\frac{A}{2} |\nabla v|^2$$

has units of energy density.

Similarly,

$$[B \mu \nu]$$

must possess identical dimensions, giving

$$[B] = \frac{\text{J}}{\text{kg}}.$$

Thus every contribution to the functional has units of

$$\text{J m}^{-3},$$

ensuring dimensional consistency throughout the derivation.

3.4 Assumptions Used

The derivation presented in this paper relies upon the following assumptions:

1. The Higgs field behaves as a continuous isotropic medium.
2. The depletion field is small,

$$\nu \ll 1.$$

3. Matter couples linearly to the depletion field.
4. Time-dependent effects are neglected.
5. The field configuration minimizes the total energy.

These assumptions define the domain of validity of the present weak-field formulation.

4. Derivation of the Weak-Field Equation

4.1 Variational Principle

The Higgs Field Bubble Theory assumes that the equilibrium configuration of the Higgs field minimizes the total field energy. Consequently, the physically realised depletion field is the function

$$v(\mathbf{x})$$

which makes the energy functional stationary under all infinitesimal variations.

The energy functional derived in the previous section is

$$E[v] = \int \left[\frac{A}{2} |\nabla v|^2 - B\mu(\mathbf{x})v \right] d^3x.$$

We now seek the function

$$v(\mathbf{x})$$

for which

$$\delta E = 0.$$

4.2 Variation of the Functional

Consider a small variation

$$v \rightarrow v + \epsilon\eta,$$

where

- ϵ is infinitesimal,
- $\eta(\mathbf{x})$ is an arbitrary smooth function that vanishes on the outer boundary.

Substituting into the energy functional gives

$$E(\epsilon) = \int \left[\frac{A}{2} |\nabla(v + \epsilon\eta)|^2 - B\mu(v + \epsilon\eta) \right] d^3x.$$

Differentiating with respect to ϵ ,

$$\frac{dE}{d\epsilon} = \int [A \nabla v \cdot \nabla \eta - B \mu \eta] d^3x.$$

The equilibrium condition requires

$$\frac{dE}{d\epsilon} \big|_{\epsilon=0} = 0.$$

4.3 Integration by Parts

Applying Green's identity,

$$\int \nabla v \cdot \nabla \eta d^3x = - \int (\nabla^2 v) \eta d^3x + \oint \eta \frac{\partial v}{\partial n} dS.$$

Because the variation vanishes at the boundary,

$$\eta = 0,$$

the surface integral disappears.

The remaining expression becomes

$$\delta E = \int [-A \nabla^2 v - B \mu] \eta d^3x.$$

Since

$$\eta$$

is completely arbitrary, the integrand itself must vanish.

Therefore,

$$-A \nabla^2 v - B \mu = 0.$$

or

$$\nabla^2 v = -\frac{B}{A} \mu.$$

This is the governing field equation of the weak-field HFBT.

5. Recovery of Newtonian Gravity

5.1 Identifying the Gravitational Potential

To compare directly with classical gravity, define the gravitational potential

$$\Phi = -C\nu,$$

where C is a proportionality constant to be determined.

Applying the Laplacian,

$$\nabla^2 \Phi = -C \nabla^2 \nu.$$

Using the field equation,

$$\nabla^2 \Phi = C \frac{B}{A} \mu.$$

The classical Poisson equation is

$$\nabla^2 \Phi = 4\pi G \mu.$$

Agreement therefore requires

$$\boxed{\frac{CB}{A} = 4\pi G}$$

which fixes the relationship between the HFBT constants.

Thus

$$\boxed{C = \frac{4\pi G A}{B}}$$

and the HFBT depletion field reproduces the Newtonian potential exactly.

5.2 Point Mass Solution

For a point mass,

$$\mu(\mathbf{x}) = M\delta(\mathbf{x}),$$

the Poisson equation has the familiar solution

$$\Phi(r) = -\frac{GM}{r}.$$

Consequently,

$$v(r) = \frac{B}{4\pi GA} \frac{GM}{r}.$$

Thus the Higgs depletion decreases as

$$\boxed{v(r) \propto \frac{1}{r}}$$

outside the source.

The Higgs field therefore approaches its undisturbed density asymptotically as distance from the mass increases.

5.3 Gravitational Acceleration

The gravitational acceleration follows directly from the potential,

$$\mathbf{g} = -\nabla\Phi.$$

Using the HFBT definition,

$$\mathbf{g} = C\nabla v.$$

Substituting the point-mass solution gives

$$\mathbf{g} = -\frac{GM}{r^2} \hat{\mathbf{r}},$$

recovering the inverse-square law exactly.

This result demonstrates that, within the weak-field approximation, Newtonian gravity is not postulated but emerges naturally from spatial gradients in the Higgs-field depletion.

5.4 Physical Interpretation

The mathematical derivation leads to a clear physical picture.

Matter creates a localised reduction in the equilibrium Higgs-field density. This depletion is greatest within the matter distribution and decreases smoothly with increasing distance. The surrounding Higgs field responds by establishing a continuous density gradient extending throughout space.

Within HFBT, gravity is identified with this gradient. Objects accelerate toward regions of increasing Higgs-field depletion because this motion follows the direction of decreasing Higgs-field energy determined by the variational principle. The field seeks to minimise its total energy. The familiar Newtonian inverse-square law therefore emerges as the macroscopic manifestation of this equilibrium process.

Unlike General Relativity, which attributes gravity to spacetime curvature, HFBT interprets gravity as a response of a physical medium whose density varies continuously around matter. In the weak-field limit, both descriptions produce identical observable predictions, although they differ in their underlying physical interpretation.

6. Light Propagation in a Higgs Density Gradient

6.1 Propagation Speed

In HFBT, light is treated as a wave propagating through the Higgs medium. If the local Higgs-field density is reduced, the local propagation speed is also reduced.

For the weak-field regime, assume the effective light speed takes the form

$$c_H(\mathbf{x}) = c \left(1 + \frac{\Phi(\mathbf{x})}{c^2} \right),$$

where $\Phi < 0$ near a gravitating mass. Therefore,

$$c_H < c.$$

Using

$$\Phi = -C\nu,$$

this may also be written as

$$c_H(\mathbf{x}) = c \left(1 - \frac{C\nu(\mathbf{x})}{c^2} \right).$$

Thus, in HFBT, reduced Higgs density corresponds to reduced local wave-propagation speed.

6.2 Effective Refractive Index

Define the effective refractive index

$$n_H(\mathbf{x}) = \frac{c}{c_H(\mathbf{x})}.$$

For weak fields,

$$\left| \frac{\Phi}{c^2} \right| \ll 1,$$

so

$$n_H = \frac{1}{1 + \Phi/c^2} \approx 1 - \frac{\Phi}{c^2}.$$

Since $\Phi < 0$, this gives

$$n_H > 1.$$

Thus, a gravitational field behaves optically like a weak refractive medium whose refractive index increases toward the mass.

6.3 Optical Path Length in a Depleted Higgs Medium

The reduction in propagation speed is not the only consequence of Higgs-field depletion.

Within HFBT, the Higgs field constitutes the physical medium through which electromagnetic waves propagate. A local reduction in field density therefore alters both the wave's propagation speed and the medium's effective spatial scale.

Let the physical path element in the undisturbed Higgs field be

$$ds.$$

In the weak-field approximation, we assume that Higgs-field depletion modifies this path element according to

$$ds_H = ds \left(1 - \frac{\Phi}{c^2} \right),$$

were

$$\left| \frac{\Phi}{c^2} \right| \ll 1.$$

The corresponding travel time, therefore

$$dt = \frac{ds_H}{c_H}.$$

Using the weak-field propagation speed

$$c_H = c \left(1 + \frac{\Phi}{c^2} \right),$$

gives

$$dt = \frac{ds \left(1 - \frac{\Phi}{c^2}\right)}{c \left(1 + \frac{\Phi}{c^2}\right)}.$$

Retaining only first-order terms,

$$dt \approx \frac{ds}{c} \left(1 - \frac{2\Phi}{c^2}\right).$$

The corresponding effective refractive index becomes

$$\boxed{n_H = 1 - \frac{2\Phi}{c^2}}$$

which is the optical index used throughout the remainder of this paper.

This result has a simple physical interpretation.

The first contribution arises from the reduction in the local propagation speed caused by Higgs-field depletion.

The second contribution arises because the depletion also changes the effective optical path length through the Higgs medium.

Together, these effects produce the complete first-order refractive index required to recover the observed weak-field gravitational lensing and Shapiro time delay. A derivation of this constitutive relation from the full nonlinear Higgs-field equations is left for future work.

7. Refraction and Gravitational Lensing

7.1 Snell-Type Refraction

At an interface between two regions of different Higgs density, the propagation speed changes. The wavefront, therefore, refracts according to

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

In HFBT, this is interpreted as ordinary refraction caused by a gradient in Higgs-field density.

In a continuously varying field, the discrete Snell relation becomes a continuous bending of the light path toward regions of larger refractive index, equivalent to regions of greater Higgs depletion.

7.2 Weak-Field Light Deflection

For a point mass,

$$\Phi(r) = -\frac{GM}{r}.$$

Therefore,

$$n_H(r) \approx 1 + \frac{GM}{rc^2}.$$

The transverse gradient in refractive index bends the ray toward the mass. To leading order, the deflection angle has the form

$$\alpha \sim \int \nabla_{\perp} n_H dz.$$

For a ray passing the mass with impact parameter b ,

$$r^2 = b^2 + z^2.$$

Using

$$n_H - 1 = \frac{GM}{c^2 \sqrt{b^2 + z^2}},$$

The transverse gradient gives

$$\nabla_{\perp} n_H = -\frac{GMb}{c^2 (b^2 + z^2)^{3/2}}.$$

Integrating along the path,

$$|\alpha| = \frac{GMb}{c^2} \int_{-\infty}^{+\infty} \frac{dz}{(b^2 + z^2)^{3/2}}.$$

Since

$$\int_{-\infty}^{+\infty} \frac{dz}{(b^2 + z^2)^{3/2}} = \frac{2}{b^2},$$

we obtain

$$\alpha = \frac{2GM}{bc^2}.$$

This is the Newtonian optical deflection term.

However, the full observed weak-field gravitational deflection is

$$\alpha_{\text{GR}} = \frac{4GM}{bc^2},$$

which contains an additional factor of two.

In HFBT this means that the effective refractive index must include both:

1. the reduction of propagation speed due to Higgs depletion, and
2. the associated spatial deformation of the propagation geometry.

Accordingly, the weak-field optical index should be written as

$$n_H(r) \approx 1 - \frac{2\Phi(r)}{c^2}$$

or

$$n_H(r) \approx 1 + \frac{2GM}{rc^2}.$$

Then the same integration gives

$$\alpha = \frac{4GM}{bc^2},$$

matching the standard weak-field result.

7.3 Interpretation of the Factor of Two

This factor is important.

A scalar slowing of light alone gives only half the observed deflection. This is the same limitation encountered in simple Newtonian emission or refractive models of gravity.

Therefore, HFBT must not claim that light bending comes only from reduced local light speed. To reproduce the observed weak-field result, the Higgs-density gradient must affect both:

wave speed

and

effective propagation geometry.

Equivalently, the weak-field optical metric must correspond to the standard post-Newtonian form.

This is not a weakness of the theory; it is a necessary consistency condition. It shows that the weak-field HFBT optical sector must reproduce the full post-Newtonian index

$$n_H \approx 1 - \frac{2\Phi}{c^2}.$$

8. Gravitational Time Delay

8.1 Travel Time Through the Medium

The travel time of light through a spatially varying Higgs medium is

$$t = \int \frac{ds}{c_H}.$$

Equivalently,

$$t = \frac{1}{c} \int n_H ds.$$

Using the weak-field index

$$n_H \approx 1 - \frac{2\Phi}{c^2},$$

The excess time delay relative to flat propagation is

$$\Delta t = -\frac{2}{c^3} \int \Phi ds.$$

For a point mass,

$$\Phi(r) = -\frac{GM}{r},$$

so

$$\Delta t = \frac{2GM}{c^3} \int \frac{ds}{r}.$$

This is the weak-field Shapiro-delay form.

8.2 Physical Meaning

In HFBT, Shapiro delay occurs because light passing near a mass propagates through a region of increased Higgs depletion. The local effective refractive index rises, increasing the travel time.

Thus, the gravitational time delay is interpreted as an optical path-length effect in the Higgs medium.

This reproduces the standard weak-field result while giving it a direct physical interpretation in terms of field density.

9. Domain of Validity

The formulation presented in this paper has been developed specifically for weak, static gravitational fields. Throughout the derivation, several simplifying assumptions have been made.

The present theory assumes:

- weak Higgs-field depletion,

$$v \ll 1,$$

- static matter distributions,

- linear coupling between matter and the Higgs field,
- isotropic Higgs-medium properties,
- small-angle optical approximations for light propagation.

Within these assumptions the resulting equations recover the standard Newtonian gravitational potential together with the accepted weak-field expressions for gravitational lensing and gravitational time delay.

The present formulation is **not** intended to describe:

- strong gravitational fields,
- relativistic collapse,
- black-hole interiors,
- nonlinear Higgs dynamics,
- quantum gravitational effects,
- cosmological evolution.

These problems require extension of the present theory beyond the linear approximation developed here.

The purpose of this work is therefore not to provide a complete gravitational theory, but to establish the classical weak-field limit upon which more general HFBT formulations may be constructed.

10. Discussion

The principal objective of this paper has been to investigate whether gravity can be interpreted as an emergent phenomenon arising from a continuous Higgs-field medium rather than as either a fundamental force or purely as spacetime curvature.

Starting from a physically motivated energy functional, a scalar Higgs depletion field has been introduced. Application of the variational principle yields a Poisson-type field equation whose solutions reproduce the Newtonian gravitational potential exactly within the weak-field approximation.

This result demonstrates that the familiar inverse-square law need not be introduced as a fundamental postulate. Instead, it emerges naturally from the equilibrium behaviour of a continuous physical medium.

The optical sector of the theory follows from the same depletion field. Variations in Higgs density modify the local propagation properties of electromagnetic waves, producing an effective refractive index. When expressed in its weak-field form, this reproduces the standard results for gravitational lensing and the Shapiro time delay.

Within the assumptions of the present weak-field approximation, the mathematical correspondence with the standard weak-field predictions is complete. The distinction lies not in the observable predictions but in the proposed physical interpretation.

General Relativity describes gravity through the geometry of spacetime.

HFBT interprets the same weak-field phenomena as arising from density gradients in a physical Higgs medium.

At the present stage, there is no experimental evidence that distinguishes these two interpretations in the weak-field regime. Consequently, the present work should be regarded as an alternative physical formulation that reproduces known classical results while providing a possible pathway toward a unified field description.

One important consequence of this interpretation is that gravity and particle structure become aspects of the same underlying medium. Matter corresponds to localised Higgs depletion, while gravitation represents the large-scale response of the surrounding field to those depletions.

Although the present paper considers only the density sector of HFBT, subsequent work suggests that additional physical phenomena—including electric charge, magnetism, spin, and quantum behaviour—may arise from a complementary phase field coupled to the depletion field at particle boundaries. The weak-field formulation presented here, therefore, serves as the classical gravitational foundation for the broader HFBT programme currently under development.

11. Conclusion

A weak-field formulation of the Higgs Field Bubble Theory has been developed from a physically motivated energy principle.

The principal results are:

1. Matter is represented by a localised depletion of a continuous Higgs field.
2. A scalar depletion field is derived using an energy minimisation principle.
3. The resulting Euler–Lagrange equation reduces to a Poisson equation in the weak-field limit.
4. The Newtonian gravitational potential and inverse-square law are recovered exactly.
5. Electromagnetic wave propagation through the Higgs medium leads naturally to an effective refractive-index description.
6. The weak-field optical formulation reproduces the accepted expressions for gravitational lensing and gravitational time delay.

These results demonstrate that a continuous Higgs medium provides a mathematically consistent alternative interpretation of weak-field gravitational phenomena.

The present work intentionally remains within the classical weak-field regime. Extensions to nonlinear gravity, strong gravitational fields, black-hole physics, and quantum phenomena remain subjects of ongoing investigation.

If these extensions can be developed while preserving the successful weak-field results established here, HFBT may provide a unified physical description linking matter, gravity, optics, and eventually quantum behaviour through the dynamics of a single underlying Higgs medium.

Author Contributions

All physical concepts, theoretical developments, mathematical derivations, and interpretations presented in this work are the original research of the author.

AI-assisted tools were used during manuscript preparation to support mathematical consistency checking, document organisation, language refinement, and editorial presentation. The author reviewed and verified all derivations and accepts full responsibility for the scientific content of this paper.

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Author Note

AI-assisted tools were used during the preparation of this manuscript to support mathematical consistency checks, document organisation, editorial revision, and language refinement. All theoretical concepts, physical interpretations, derivations, and conclusions presented in this work are the original research of the author. The author has reviewed all mathematical developments and accepts full responsibility for the scientific content of the manuscript.